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HCN-003-001544

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October - 2017

S - 503 : Statistics

(Statistical Inference)

(New Course)

Faculty Code : 003
Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Q.1 carries 20 marks.
(2) Q. 2 and 3 each carries 25 marks.
(3) Students are allowed to use their own scientific calculator.

1 Fill in the blanks and short questions : (Each 1 mark) **20**

(1) An estimator T_n which is most concentrated about parameter θ is the __________ estimator.

(2) Estimation is __________ if we have a purposive sample.

(3) If $X_1, X_2, X_3, ...X_n$ be a random sample, the expression $\sum \frac{x_i}{n}$ is an __________

(4) A single value of an estimator for a population parameter θ is called its __________ estimate.

(5) If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to __________.

(6) If $T_n = t_n(X_1, X_2, X_3, ...X_n)$, an estimator of $\tau(\theta)$, is such that $\lim_{n\to\infty}\left[T_n - \tau(\theta)\right]^2 = 0$, T_n is said to be __________ consistent.

(7) An estimator is efficient if its variance is __________ than the variance of any other estimator.

(8) If a statistic $T = t(X_1, X_2, X_3, ...X_n)$ provides as much information as the random sample $X_1, X_2, X_3, ...X_n$ could provide, then T is a __________.

(9) If $f(x;\theta)$ is a family of distributions and $h(x)$ is any statistic such that $E\left[h(x)\right] = 0$, then $f(x;\theta)$ is called __________.

(10) A maximum likelihood estimate is not necessarily __________.

(11) If an MVB unbiased estimator exists, __________ estimator provides it.

(12) Maximum likelihood estimator $\frac{\sum(x_i - \bar{x})^2}{n}$ of the variance σ^2 of a normal density $f\left(x;\mu,\sigma^2\right)$ is a __________

(13) For a rectangular distribution $\frac{1}{(\beta-\alpha)}$ the maximum likelihood estimates of α and β are__________ and __________ respectively.

(14) The estimation of a parameter by the method of minimum Chi-square utilizes __________ statistic.

(15) The estimators obtained by the method of minimum Chi-square and maximum likelihood estimator are __________.

(16) If $E\left(T_n\right) > \theta,$ the parameter value T_n is said to be __________.

(17) An unbiase estimator is not necessarily __________.

(18) If mean $\bar{x}$ of a sample drawn from a Normal population is a maximum likelihood estimator, $\bar{x}$ is a __________ estimator of population mean.

(19) Sample mean is an __________ and __________ estimate of population mean.

(20) If T_1 and T_2 are two MVU estimator for $T(\theta)$, then __________

2 (A) Write the answers of any **Three** : (Each of 2 marks) **6**

(1) Define Consistency

(2) Show that $\sum x_i$ is a sufficient estimator of θ for Geometric distribution.

(3) Define Complete family of distribution

(4) Define Uniformly Most Powerful Test (UMP test)

(5) Obtain an unbiased estimator of θ by for the following distribution $f(x:\theta)=\frac{1}{\theta};0\le x<\theta$

(6) Define Efficiency

(B) Write the answers any of **Three** : (Each of 3 marks) **9**

(1) Let $x_1, x_2, x_3, \ldots, x_n$ be random sample taken from $N\left(\mu,\sigma^2\right)$ then find sufficient estimator of μ and σ^2.

(2) $\frac{\overline{x}}{n}$ is a consistent estimator of p for Binomial distribution.

(3) Obtain MVUE of parameter θ for Poisson distribution.

(4) A is more efficience than B then prove that $Var(A) + Var(B - A) = Var(B)$

(5) Given a random sample $x_1, x_2, x_3, ... x_n$ from distribution with p.d.f. $f(x;\theta) = \frac{1}{\theta}; 0 \le x \le \theta$, Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x : x \ge 0.8\}$.

(6) Obtain Operating Characteristic (OC) function of SPRT.

(C) Write the answers of any **Two** : (Each of 5 marks) **10**

(1) State Neyman-Pearson Lemma and prove it.

(2) Estimate α and β in the case of Gamma distribution by the method of moments

$$f(x; \alpha, \beta) = \frac{\alpha^{\beta}}{\Gamma\beta} e^{-\alpha x} x^{\beta - 1}; x \ge 0, \alpha \ge 0$$

(3) Construct SPRT of Poisson distribution for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$. Also obtain OC function of SPRT.

(4) Given a random sample $x_1, x_2, x_3, ... x_n$ from distribution with p.d.f. $f(x;\theta) = \theta e^{-\theta x}$; $0 \le x \le \infty, \theta > 0$ Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(5) Obtain Likelihood Ratio Test :
Let $x_1, x_2, x_3, ... x_n$ random sample taken from $N(\mu, \sigma^2)$. To test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \ne \sigma_0^2$.

3 (A) Write the answers any **Three** : (Each of 2 marks) **6**

(1) Define Unbiasedness

(2) Define Sufficiency

(3) Define Minimum Variance Bound Estimator (MVBE)

(4) Define Most Powerful Test (MP test)

(5) Define ASN function of SPRT

(6) Show that sample mean is more efficient than sample median for Normal distribution.

(B) Write the answer any **Three** : (Each 3 marks) **9**

(1) Obtain unbiased estimator of $\frac{kq}{p}$ of Negative Binomial distribution.

(2) Obtain an unbiased estimator of population mean of $\aleph^2$ distribution.

(3) Prove that $E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)$

(4) Obtain estimator of θ by method of moments in the following distribution $f(x;\theta) = \theta x^{\theta-1}; 0 \le x \le 1$ *If*

(5) Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$ in the case of Poisson distribution with parameter λ.

(6) Let P be the probability that coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 6 times and H_0 is rejected if more than 4 head are obtained, Find the probability of type-I error, type-II error and power of test.

(C) Write the answers any **Two** : (Each of 5 marks) **10**

(1) State Crammer-Rao inequality and prove it.

(2) Obtain MVBE of σ^2 for Normal distribution.

(3) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation p, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination ?

(4) For the double Poisson distribution. $P(X = x) = \frac{1}{2}\frac{e^{-m_1}{m_1}^x}{x!} + \frac{1}{2}\frac{e^{-m_2}{m_2}^x}{x!}; 0, 1, 2, \ldots$ Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - (\mu_1')^2}$

(5) Obtain OC function for SPRT of Binomial distribution for testing $H_0 : p = p_0$ against $H_1 : p = p_1 (> p_0)$